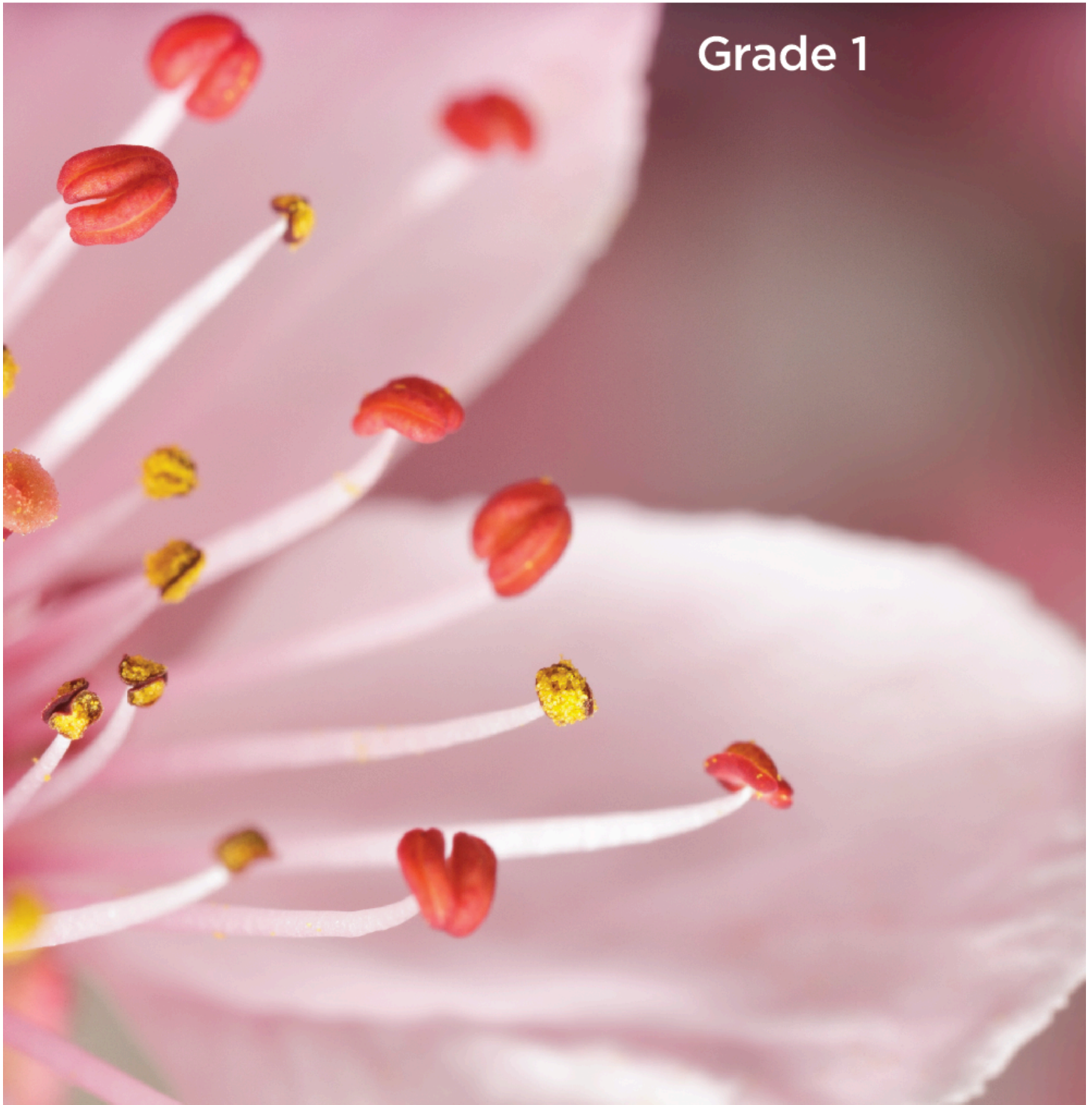


Early Mathematics

a resource for teaching young children



Grade 1

a publication of
The Charles A. Dana Center at The University of Texas at Austin

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**A Resource for Teaching Young
Children**

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The University of Texas at Austin

2012

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October 2012 release.

As always, we welcome your comments and suggestions for improvements. Please contact us at dana-txshop@utlists.utexas.edu or at the mailing address above.

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center strengthens our nation’s education systems to provide a reliable path to upward mobility for all students. Our work focuses on mathematics and science education, with an emphasis on strategies for improving student engagement, motivation, and persistence. We are dedicated to nurturing students’ intellectual passions and ensuring that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy.

We advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards. We help our partners adapt promising research to meet their local needs.

We develop innovative curricula, tools, protocols, instructional supports, and professional development systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance to school and district leadership teams, advise community colleges and states, and collaborate with national partners on work such as our Urban District Leadership Networks, Academic Youth Development project, and Advanced Mathematical Decision Making course.

We have significant experience and expertise in the following:

- Standards development and implementation, systemic reform, and district capacity building
- Education leadership, instructional coaching, and teaching
- K–14 course design and development, learning networks, and programs for bridging critical transitions
- Research, content development, and publishing

The Center was founded in 1991 at The University of Texas at Austin. Our staff of nearly 80 researchers and education professionals has worked with dozens of school systems in nearly 20 states and with 90 percent of Texas’s more than 1,000 school districts. We are committed to ensuring that the accident of where a child attends school does not limit the academic opportunities he or she can pursue. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

About the Common Core State Standards for Mathematics

This resource is aligned to the Common Core State Standards for Mathematics.

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About the development of this resource

This new revised and expanded edition of *Early mathematics: A resource for teaching young children* consists of materials for 20 sessions for each of four grades—prekindergarten, kindergarten, grade 1, and grade 2—for a total of 80 sessions. We were able to develop these materials because of a generous December 2010 grant from the **Noyce Foundation**.

First edition (2011)

Twenty of these sessions (10 for prekindergarten and 10 for second grade) were initially developed in spring and summer 2011 by early mathematics education experts Brian Mowry (prekindergarten) and Carolyn Moore (second grade), and reviewed in summer and early fall 2011 by ACE: A Community for Education leaders Chetan Makan and Mary Ellen Isaacs, both of whom are experts in designing and implementing early childhood tutoring programs that can be implemented at scale. The materials were also reviewed by Patti Bridwell, who has expertise in professional supports for teachers and tutors.

This first edition was released in fall 2011 as a proof-of-concept resource titled *Early mathematics: Resources for tutoring young children*. These initial 20 sessions were field-tested in fall 2011 by tutors from the Dana Center’s ACE: A Community for Education (www.utdanacenter.org/ace) program in Austin, Texas (prekindergarten sessions), and by tutors from Experience Corps (www.experiencecorps.org) in Philadelphia, Pennsylvania (second-grade sessions).

Second edition (2012)

A key finding from the fall 2011 proof-of-concept field testing was that the material as written was probably too complex for paraprofessionals (e.g., tutors) to deliver, but that it could be very effective if delivered by classroom teachers. Based on this feedback, we have substantially revised the initial 20 sessions for this new edition, changing the intended users of this resource from paraprofessionals to classroom teachers.

All 80 sessions are built on recommendations in the 2009 National Research Council report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Committee on Early Childhood Mathematics; Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, editors). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D.C: The National Academies Press.

In particular, these session materials speak to the recommendation that:

Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with *more mathematics instruction time devoted to number* than to other topics.

Accordingly, our materials focus primarily on number.

About the Noyce Foundation

The Noyce Foundation¹ aims to help young people become curious, thoughtful, and engaged learners. The Noyce Foundation focuses on a few key areas:

- improving the teaching of math and science in public schools;
- developing leadership to support student achievement;
- supporting education policy and research; and
- expanding opportunities for students to experience hands-on science in out-of-school settings.

The Noyce family created the Noyce Foundation in 1990 to honor the memory and legacy of Dr. Robert N. Noyce, cofounder of Intel and inventor of the integrated circuit—which fueled the personal computer revolution and gave Silicon Valley its name.

Although he was an individual of daunting talents and intellect who was honored by two presidents as well as by his academic and industry peers around the world, Bob Noyce also remained a humble and approachable man who believed fervently in democracy. In everything the Noyce Foundation undertakes, it remains committed to promoting the qualities that Bob Noyce embodied: optimism, creativity, risk taking, and determination.

In recognition of Bob’s concern about the narrowing pipeline of students interested in—and committed to—science-related careers, the Noyce Foundation has focused on mathematics, science, and associated work in research and policy. Much of the Foundation’s focus has been on improving instruction in mathematics, science, and early literacy in public schools.

As schools began to intensify their focus on math and literacy in response to No Child Left Behind—leaving science behind in the process—the Noyce Foundation emphasized support for out-of-school science programs that show promise of sustaining and engaging student interest through middle school, a time when students tend to make critical decisions about what subjects they want to pursue in the future. The Noyce Foundation informal science initiative includes support for leadership development in science centers.

For more information about the Noyce Foundation, visit its website at www.noycefdn.org. For more information about the Silicon Valley Mathematics Initiative, see www.svmimac.org.

¹ This description of the Noyce Foundation’s mission and history was adapted from content retrieved from its homepage (www.noycefdn.org) and its About Us page (www.noycefdn.org/aboutus.php) on October 9, 2012.

Acknowledgments

With special thanks . . .

The Dana Center thanks the Noyce Foundation for its generous support of this project. The Noyce family created the Noyce Foundation in 1990 to honor the memory and legacy of Dr. Robert N. Noyce, cofounder of Intel and inventor of the integrated circuit—which fueled the personal computer revolution and gave Silicon Valley its name. For more information about the Noyce Foundation, visit its website at www.noycefdn.org.

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Introduction

Background

Early Mathematics—A Resource for Teaching Young Children provides a series of instructional tasks, aligned with the Common Core State Standards for Mathematics, that teachers can use to instruct children in prekindergarten, kindergarten, grade 1, and grade 2. The complete resource includes content for 20 sessions for each of these four grade levels.

The tasks were developed for whole-class instruction with some small-group work, but they are also easily adaptable for tutoring sessions. The estimated timeframe for each session is as follows:

Grade level	Estimated time per session
Prekindergarten	30 minutes
Kindergarten	45 minutes
Grade 1	45 minutes
Grade 2	45 minutes

Most sessions have a literature focus to draw children into the content and/or to keep them connected to a context.

These session materials do not provide everything a child needs to know about a given topic, such as *number*. Rather, each session provides a series of instructional tasks to help you teach selected content and practices described in the Common Core State Standards for Mathematics. You should feel free to modify the sessions as appropriate to meet the individual needs of children in your classroom.

Alignment

We have embedded key Common Core State Standards for Mathematical Practice in each session to help bring out crucial ideas. In most sessions, though, additional Standards for Mathematical Practice beyond those selected may also be relevant.

We chose the content for these sessions based on what content we believe will have the most significant effect on student learning. The language below is drawn from the National Council of Teachers of Mathematics 2006 publication, *Curriculum Focal Points for Prekindergarten Through Grade Eight Mathematics: A Quest for Coherence* (prekindergarten) and the Common Core State Standards for Mathematics (kindergarten onward).

Prekindergarten

- (1) developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison.

Kindergarten

- (1) representing, relating, and operating on whole numbers, initially with sets of objects;

Grade 1

- (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
- (2) developing understanding of whole number relationships and place value, including grouping in tens and ones;
- (3) developing understanding of linear measurement and measuring lengths as iterating length units.

Grade 2

- (1) extending understanding of base-ten notation;
- (2) building fluency with addition and subtraction.

Structure

Each session is divided into three instructional formats—**Activate**, **Engage**, and **Develop**.

The **activate** portion introduces the content in the session and objectives that will be developed in the forthcoming session. In prekindergarten and kindergarten, this section can often occur as a part of the morning circle routine (e.g., calendar, morning message), or it can serve as a transition activity that incorporates songs, movement, and other instructional activities developed to capture the interest and attention of younger students with emerging attention spans.

Then children will **engage** in the content through an activity centered on the content and practices in the standard(s) being addressed. In prekindergarten and kindergarten, this time is spent mostly in whole (or large) group so that the teacher can model the mathematical thinking that children will apply in the Develop section. For younger children, keep in mind that whole-group sessions are designed to last no longer than 20 minutes.

Each session ends with **develop**, which provides children an opportunity to share and analyze their understandings and/or methods. In prekindergarten and kindergarten, the activities in this section can take place during centers, small group, or math station time. Throughout, the role of the teacher will primarily be to ask probing questions to help children make sense of the content in the session.

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A Resource for Teaching Young Children

Grade 1

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Session 1: Ordering Three Objects by Length

At A Glance

In this session, children explore the attribute of length and compare the length of two objects directly and a third indirectly. Children describe how they know when an item is longer or longest and shorter or shortest. They create an arrangement that shows the relationships among the items in the arrangement. This may require a systematic strategy and draws on a child's reasoning to determine that if Item A is longer than Item B and Item B is longer than Item C, then Item A is longer than Item C (transitivity).

Common Core State Standards focus

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.

MP 2 Reason abstractly and quantitatively.

Materials

- *Big & Little* by Steve Jenkins
- A bag of items you can compare lengths for each group of children
 - an unsharpened pencil
 - an unused crayon
 - three straws, with one cut to be 2 inches, one cut to be 6 inches, and one left uncut but 8 inches or longer
- Transparent tape
- Construction paper



Activate

Show children the cover of *Big & Little* by Steve Jenkins.

- Ask, ***“What do you think this story is about?”***
- ***“What does the word big mean? How do you know if something is big?”***
- ***“What does the word little mean? How do you know if something is little?”***

Read *Big & Little*.

- ***“What are some other words we can use to describe how big or little something is?”***
(long, short, tall, wide, thin)

Turn back to the page with the snakes.

- Say, ***“What do you notice about these two snakes? Yes, the rock python is longer than the coral snake. Or you could say the coral snake is shorter than the rock python. What do we mean when we say longer and shorter? What are we measuring?”***



Engage

Organize children into pairs. Give each pair a bag of items to measure length.

- Say, ***“Take the pencil and crayon from your bag. Directly compare them. What do you notice?”***
- ***“Which is longer? Which is shorter? How do you know?”***
- ***“Take the three straws out of your bag and place the pencil back in the bag.”***
- ***“Use the crayon to find a straw that is shorter than the crayon. How do you know it’s shorter than the crayon?”*** (Direct comparison)
- ***“Is the straw you found shorter or longer than the pencil? How did you know without comparing the straw to the pencil?”*** (Indirect comparison—If the straw is shorter than the crayon and the crayon is shorter than the pencil, then the straw must also be shorter than the pencil.)

- ***“Remove the pencil from the bag and place the crayon in the bag.”***
- ***“Use the pencil to find a straw that is longer than the pencil. How do you know it’s longer than the pencil?”***
- ***“Is the straw you found shorter or longer than the crayon? How did you know without comparing the straw to the crayon?”***
- ***“Place the crayon in the bag with the pencil.”***
- ***“Compare and order the three straws.”***
- ***“Tape the straws on the construction paper in a way that clearly shows them in order by length.”***



Develop

Have one pair of children share their arrangement of straws. Ask questions such as the following:

- How did you decide which straw was longer and longest?
- How did you decide which straw was shorter and shortest?
- Why did you organize them on the paper this way?

Repeat with other groups.



Session 2: Iterating Length Units

At A Glance

In this session, children connect the comparative length they did in Session 1 to quantifying length. They use an object with less length to measure an object that is longer. Children must think carefully about where they start and end measuring. They must also consider the importance of not leaving spaces between or overlapping the objects being used for measuring. This session is critical as it builds an obvious relationship between length and quantities. Consequently, children may benefit from more experience with iterating before making connections to addition and subtraction problem types in Sessions 3–9.

Materials

- *Big & Little* by Steve Jenkins
- Square tiles or paper clips that have a length of 1 inch
- Masking tape
- Unsharpened pencils
- Blackline master, “Tails”

Common Core State Standards focus

Measurement and Data

Measure lengths indirectly and by iterating length units.

- 1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

MP 2 Reason abstractly and quantitatively.

MP 6 Attend to precision.



Activate

Before this session, make five or six “kangaroo tails” in different parts of the room. To make the tail, simply place a strip of masking tape carefully cut 37.5 inches in length on the floor or a table.

Remember the Siamese cat and the Siberian tiger in *Big & Little*.

- Ask, **“How could we describe the length of the tail of the Siberian tiger?”** (long, longer than the Siamese cat’s tail, longer than my pencil)
- **“How about the Siamese cat’s tail?”** (longer than my pencil, but shorter than my arm)

Turn to the page with the Nile crocodile and the African chameleon.

- **“How about these two tails? How would you describe their length?”**

Turn to the page with the red kangaroo.

- **“Let’s imagine that the kangaroo is here, asleep. So he is very still. His tail is stretched straight out.”**

Show them the line on the floor or table used to represent the kangaroo tail.

- **“How could we use these pencils to measure the kangaroo’s tail?”** (start at one end, line the pencils up with no gaps or overlaps)

Have each group move to a kangaroo tail (masking tape). Each child in the group should have a turn measuring the kangaroo’s tail. (**Note:** A typical unsharpened pencil is 7.5 inches, so it should take five pencils.)

- **“If we had used crayons instead of pencils on the kangaroo tail, how would our results (our findings) have changed?”** (It would take more crayons than pencils to equal the length of the kangaroo tail.)



Engage

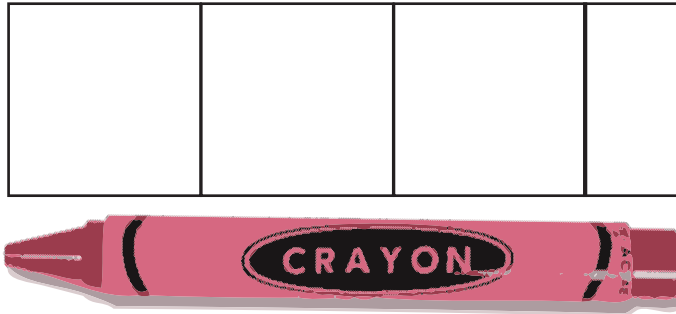
Give children the blackline, “Tails.”

- Children use square tiles to measure each tail and record their findings.
- Remind children that the tiles must be lined up carefully with no gaps or overlaps.

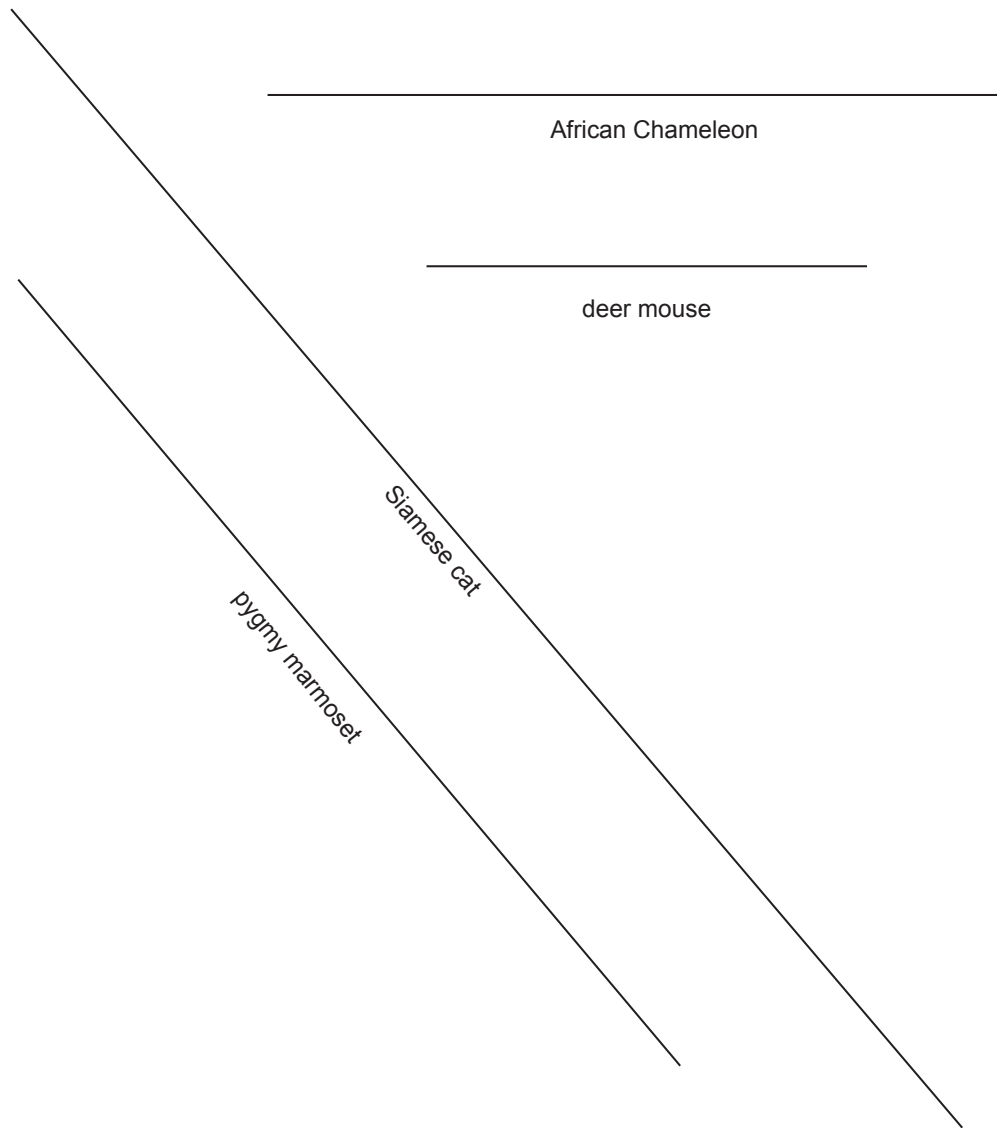
Develop

In a whole group, have children describe their results and what they did to achieve them. Discuss any discrepancies.

- ***“If we had used crayons instead of square tiles on the animal tails how would our results (our findings) have changed?”*** (It would take fewer crayons than square tiles to equal the length of the animal tails.)
- ***“If you were going to tell a friend how to accurately measure using square tiles or another object, what would you say?”***



Tails



Session 3: Add-To/Take-From Problems (Result Unknown)

At A Glance

In this session, children represent and solve add-to and take-from problems where the result is unknown ($A + B = \square$ and $C - B = \square$). They use linking cubes to represent the situation and then compose or decompose the sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Add-To/ Take-From Problems (Result Unknown)”

$$A + B = \square$$

$$C - B = \square$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

Children need 10 linking cubes each. It is important to have a way for all children to see what you are doing with the cubes (e.g., through the use of a document camera).

- Say, ***“We have been comparing and measuring tails. Now we are going to help some children build make-believe tails with linking cubes.”***

Post the following problem:

Marco made a tail with 3 cubes. Then Emma gives Marco 2 cubes to add to his tail. Now how long is Marco’s tail?

- ***“Let’s look at this problem.”***
- ***“Marco made a tail with 3 cubes. That can be our starting point. Let’s make the same tail.”***

Make a stick of three cubes as children do the same. Write the number 3.

- ***“Did we make the same size tail Marco did? How do we know?”***
- ***“Then Emma gives Marco 2 cubes to add to his tail. Something changed, didn’t it? Let’s show what Emma gave Marco.”***

Make a stick of two cubes as children do the same. Write + 2, so you now have the expression $3 + 2$.

- ***“Is that all that changed?”*** (Yes)

Write = ? to make the equation $3 + 2 = ?$.

- ***“Now how long is Marco’s tail?”***

Show the three-cube stick and two-cube stick close together as if you were making one stick, but do not connect them. (**Note:** It is important not to connect the sticks at this point because you want children to see the relationship between the two parts and the whole.)

Below the equation $3 + 2 = ?$, write the equation $3 + 2 = 5$.

- ***“Let’s look at the equations we have written. How do they describe what happened?”***
- ***“How do the parts in the equation connect to the model we have made with the linking cubes?”***



Engage

Organize children into pairs. Hand out the blackline, “Add-To/Take-From Problems (Result Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child builds the starting point with the linking cubes and then records the number that represents that starting point on their paper.
- Children confer with each other about the starting point.
- Each child then builds and represents the change and solution with a linking cubes model and by completing the equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.



$$7 - 3 = 4$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Session 4: Add-To/Take-From Problems (Change Unknown)

At A Glance

In this session, children represent and solve add-to and take-from problems where the change is unknown ($A + \square = C$ and $C - \square = A$). They use linking cubes to represent the situation and then compose or decompose the sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Add-To/ Take-From Problems (Change Unknown)”

$$A + \square = C$$

$$C - \square = A$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

Children need 10 linking cubes each. It is important to have a way for all children to see what you are doing with the cubes (e.g., through the use of a document camera).

Post the following problem:

Marco has 5 cubes on his tail. He put some more cubes on his tail. Then his tail was 9 cubes long. How many more cubes did Marco put on his tail?

- Say, ***“Let’s look at this problem.”***
- ***“What can be our starting point?”***
- ***“Marco has 5 cubes on his tail.”***

Make a stick of five cubes as children do the same. Write the number 5.

- ***“Did we make the same size tail Marco did? How do we know?”***
- ***“Then what happened? Something changed, didn’t it?”***
- ***“He put some more cubes on his tail, but we don’t know how many.”***

Write $+$ $?$, so you now have the expression $5 + ?$.

- ***“Then the tail was 9 cubes long.”***

Write $= 9$ to make the equation $5 + ? = 9$.

- ***“How does this equation represent the situation?”***
- ***“Let’s find out how many cubes Marco added.”***

Start building another stick that when added to the 5-cube stick makes 9.

- ***“Will a 5 tail and a 2 tail make a 9 tail? Why not?”***
- ***“How about a 5 tail and a 3 tail? Or a 5 tail and a 4 tail?”***

Note: It is important not to connect the cubes at this point because you want children to see the relationship between the two parts and the whole.

Below the equation $5 + ? = 9$, write the equation $5 + 4 = 9$.

- ***“Let’s look at the equations we have written. How do they describe what happened?”***
- ***“How do the parts in the equation connect to the model we have made with the linking cubes?”***



Engage

Organize children into pairs. Hand out the blackline, “Add-To/Take-From Problems (Change Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child builds the starting point with the linking cubes and then records the number that represents that starting point on their paper.
- Children confer with each other about the starting point.
- Each child then builds and represents the change and solution with a linking cubes model and by completing the equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.



$$10 - 3 = 7$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Session 5: Add-To/Take-From Problems (Start Unknown)

At A Glance

In this session, children represent and solve add-to and take-from problems where the start is unknown ($\square + B = C$ and $\square - B = A$). They use linking cubes to represent the situation and then compose or decompose the sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Add-To/ Take-From Problems (Start Unknown)”

$$\square + B = C$$

$$\square - B = A$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

Children need 10 linking cubes each. It is important to have a way for all children to see what you are doing with the cubes (e.g., through the use of a document camera).

Post the following problem:

Emma has a tail she already made. Marco gave Emma 6 cubes to put on her tail. Then Emma had 10 cubes on her tail. How many cubes did Emma have on her tail before Marco gave her some?

- Say, ***“Let’s look at this problem.”***
- ***“What can be our starting point?”***
- ***“Emma has a tail she already made, but we don’t know how many cubes were on it.”***

Write $?$.

- ***“Then what happened? Something changed, didn’t it?”***
- ***“Marco gave Emma 6 cubes to put on her tail.”***

Write $+ 6$, so you now have the expression $? + 6$.

- ***“Then Emma had 10 cubes on her tail.”***

Write $= 10$ to make the equation $? + 6 = 10$. Build a stick of 6 to represent the amount Marco gave Emma.

- ***“How does this equation represent the situation?”***
- ***“How many cubes did Emma have on her tail before Marco gave her some?”***

Start building another stick that when added to the 6-cube stick makes 10.

- ***“Will a 3 tail and a 6 tail make a 10 tail? Why not?”***

Note: It is important not to connect the cubes at this point because you want children to see the relationship between the two parts and the whole.

Below the equation $? + 6 = 10$, write the equation $4 + 6 = 10$.

- ***“Let’s look at the equations we have written. How do they describe what happened?”***
- ***“How do the parts in the equation connect to the model we have made with the linking cubes?”***



Engage

Organize children into pairs. Hand out the blackline, “Add-To/Take-From Problems (Start Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child builds the starting point with the linking cubes and then records the number that represents that starting point on their paper.
- Children confer with each other about the starting point.
- Each child then builds and represents the change and solution with a linking cubes model and by completing the equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.



$$10 - 8 = 2$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Add-To/Take-From Problems (Start Unknown)

1. Marco has a tail he already made. Emma took 8 cubes off of Marco's tail. Then Marco has 2 cubes on his tail. How many cubes did Marco have on his tail before Emma took off some cubes?

2. Zoey has a tail she already made. Paul gave Zoey 4 cubes to put on her tail. Then Zoey had 9 cubes on her tail. How many cubes did Zoey have on her tail before Paul gave her some?

3. Paul has a tail he already made. Zoey took 3 cubes off of Paul's tail. Then Paul has 5 cubes on his tail. How many cubes did Paul have on his tail before Zoey took off some cubes?

Session 6: Put-Together/Take-Apart Problems (Addend Unknown)

At A Glance

In this session, children represent and solve put-together/take-apart problems where the addend is unknown ($A + \square = C$ and $C - A = \square$). They use linking cubes to represent the situation and then compose or decompose the sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Put-Together/Take-Apart Problems (Addend Unknown)”

$$A + \square = C$$

$$C - A = \square$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

All children need different colored cubes (preferably red, blue, green, and yellow). It is important to have a way for all children to see what you are doing with the cubes (e.g., through the use of a document camera).

Post the following problem:

Marco's tail is 10 cubes long. 4 cubes are red, and the rest are blue. How many of the cubes are blue?

- Say, ***"Let's look at this problem."***
- ***"What can be our starting point?"***
- ***"Marco's tail is 10 cubes long."***

Write 10.

- ***"Nothing changed this time, but we learned something about Marco's tail."***
- ***"4 cubes are red, and the rest are blue. How many of the cubes are blue?"***

Write $10 - 4 = ?$, so you now have the equation $10 - 4 = ?$.

- ***"How would we represent that with the linking cubes?"***

Start with a stick of 10 cubes, and break off 4 cubes.

- ***"Is there another way we can represent this situation with an equation?"***
- ***"How about $4 + ? = 10$? How does this equation represent the situation?"***
- ***"How would we represent that with the linking cubes?"***

Start with a stick of 4 and determine what you can add to the stick to get 10 cubes. Below the equation $10 - 4 = ?$, write the equation $10 - 4 = 6$. Below the equation $4 + ? = 10$, write the equation $4 + 6 = 10$.

- ***"Let's look at the equations we have written. How do they describe the situation?"***
- ***"How do the parts in the equation connect to the model we have made with the linking cubes?"***



Engage

Organize children into pairs. Hand out the blackline, “Put-Together/Take-Apart Problems (Addend Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child builds the starting point with the linking cubes and then records the number that represents that starting point on their paper.
- Children confer with each other about the starting point.
- Each child then builds and represents the change and solution with a linking cubes model and by completing the equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.



$$5 + 3 = 8$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Session 7: Put-Together/Take-Apart Problems (Both Addends Unknown)

At A Glance

In this session, children represent and solve put-together/take-apart problems where both addends are unknown ($C = \square + \square$). They use linking cubes to represent the situation and then compose or decompose sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Put-Together/Take-Apart Problems (Both Addends Unknown)”

$$C = \square + \square$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

All children need different colored cubes (preferably red, blue, green, and yellow). It is important to have a way for all children to see what you are doing with the cubes (e.g., through the use of a document camera).

Post the following problem:

Emma has a red and blue tail. Her tail is 6 cubes long. How many of the cubes on her tail could be red, and how many could be blue?

- Say, ***“Let’s look at this problem.”***
- ***“How could we represent this situation with an equation?”*** ($6 = \square + \square$)
- ***“How would we represent this situation with the linking cubes?”***

Children may want to start with a stick of 6 cubes and then break it to find combinations that make 6 (decomposing). Or they may start by trying to find a combination that makes 6 (composing). Make sure that both representations come out of the discussion.

- ***“What are all the possible answers to this problem?”***

Record the equations.

- ***“Let’s look at the equations we have written. How do they describe the situation?”***
- ***“How do the parts in the equation connect to the model we have made with the linking cubes?”***

$$6 \text{ cubes} = 5 \text{ red cubes} + 1 \text{ blue cube}$$

$$6 \text{ cubes} = 4 \text{ red cubes} + 2 \text{ blue cubes}$$

$$6 \text{ cubes} = 3 \text{ red cubes} + 3 \text{ blue cubes}$$

$$6 \text{ cubes} = 2 \text{ red cubes} + 4 \text{ blue cubes}$$

$$6 \text{ cubes} = 1 \text{ red cube} + 5 \text{ blue cubes}$$

Engage

Organize children into pairs. Hand out the blackline, “Put-Together/Take-Apart Problems (Both Addends Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child determines how to represent the problem with the linking cubes and writes a corresponding equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.

$$4 \text{ cubes} = 2 \text{ green cubes} + 2 \text{ yellow cubes}$$

$$4 \text{ cubes} = 3 \text{ green cubes} + 1 \text{ yellow cube}$$

$$4 \text{ cubes} = 1 \text{ green cube} + 3 \text{ yellow cubes}$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Session 8: Compare Problems (Difference Unknown)

At A Glance

In this session, children represent and solve compare problems where the difference is unknown ($A + \square = C$ or $C - A = \square$). They use linking cubes to represent the situation and then compare sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Compare Problems (Difference Unknown)”

$$A + \square = C$$

$$C - A = \square$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

All children need 20 cubes. It is important to have a way for them to see what you are doing with the cubes (e.g., through the use of a document camera).

Post the following problem:

Marco's tail has 7 cubes. Emma's tail has 9 cubes. How many more cubes does Emma have than Marco?

- Say, ***“Let’s look at this problem.”***
- ***“How would we represent this situation with the linking cubes?”***

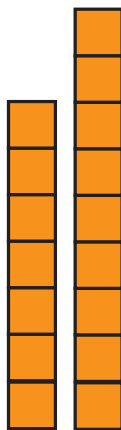
Build both a 7-cube stick and 9-cube stick because this is a compare problem. Hold them against each other so that the focus is on the difference.

- ***“How could we represent this situation with an equation?”*** ($7 + \square = 9$ or $9 - \square = 7$)
- ***“How might we solve this problem?”***

Children may start with the 7-cube stick and count up to the 9-cube stick, finding a difference of 2. Or they may start with the 9-cube stick and count down to the 7-cube stick, finding a difference of 2.

Record the equations $7 + 2 = 9$ and $9 - 2 = 7$.

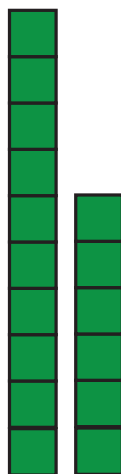
- ***“Let’s look at the equations we have written. How do they describe the situation?”***
- ***“How do the parts in the equation connect to the model we have made with the linking cubes?”***



Engage

Organize children into pairs. Hand out the blackline, “Compare Problems (Difference Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child determines how to represent the problem with the linking cubes and writes a corresponding equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.



$$10 - 6 = 4$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Compare Problems (Difference Unknown)

1. Emma's tail has 6 cubes. Marco's tail has 10 cubes. How many fewer cubes does Emma have than Marco?
2. Paul's tail has 3 cubes. Zoey's tail has 5 cubes. How many more cubes does Zoey have than Paul?
3. Zoey's tail has 5 cubes. Paul's tail has 10 cubes. How many fewer cubes does Zoey have than Paul?

Session 9: Compare Problems (Bigger/Smaller Unknown)

At A Glance

In this session, children represent and solve compare problems where the bigger is unknown ($A + B = \square$) or the smaller is unknown ($C - B = \square$ or $\square + B = C$). They use linking cubes to represent the situation and then compare sticks (called *tails* in this session) to solve. Children also represent the situation by writing an equation that mirrors their work with the linking cubes. As in the equation, the model created with the linking cubes shows a relationship between the parts and the whole. The context of the problems is thin and remains consistent so that the focus is on the problem-solving types (MP 1) and representing those problem types (MP 2) rather than on navigating a new context. Although this context continues to serve as a touchstone for learning a new problem type and connect quantities to the study of length in Sessions 1 and 2, it is advisable that children experience more problems of this type with varied contexts after this lesson.

Materials

- Linking cubes
- Blackline master, “Compare Problems (Bigger/Smaller Unknown)”

$$A + B = \square$$

$$C - B = \square$$

$$\square + B = C$$

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²

² See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.

Activate

All children need 20 cubes. It is important to have a way for them to see what you are doing with the cubes (e.g., through the use of a document camera).

Post the following problem:

Emma has 3 more cubes on her tail than Marco has on his tail. Marco has 6 cubes on his tail. How many cubes does Emma have on her tail?

- Say, ***“Let’s look at this problem.”***
- ***“How would we represent this situation with the linking cubes?”***
- ***“How could we represent this situation with an equation?”*** ($6 + 3 = ?$)
- ***“How might we solve this problem?”***

Record the equation $6 + 3 = 9$.

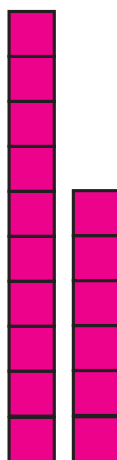
- ***“Let’s look at the equation. How does it describe the situation?”***
- ***“How do the parts in the equation connect to the model we have made with the linking cubes?”***



Engage

Organize children into pairs. Hand out the blackline, “Compare Problems (Bigger/Smaller Unknown).”

- Children read the first problem together. (**Note:** If they have difficulty reading these problems, restructure the Engage section so that you are reading each problem before pairs begin working it.)
- Each child determines how to represent the problem with the linking cubes and writes a corresponding equation. (**Note:** Children may find more than one valid equation. That is an important discovery and leads to future work as long as it can be justified with the situation and the linking cubes model.)
- Children confer with each other about how the equation matches their linking cubes model. Then they move to the next problem.



$$10 - 4 = 6$$

Develop

Organize the class into a whole group.

- Go through each problem in the blackline.
- Have children discuss and show the representations they used with the linking cubes and by writing equations for each problem.
- Continue to emphasize and ask questions about the connections among the situation, linking cubes model, and equations.

Compare Problems (Bigger/Smaller Unknown)

1. Marco has 4 fewer cubes on his tail than Emma has on her tail. Emma has 10 cubes on her tail. How many cubes does Marco have on his tail?
2. Zoey has 7 fewer cubes on her tail than Paul has on his tail. Paul has 10 cubes on his tail. How many cubes does Zoey have on her tail?
3. Paul has 8 more cubes on his tail than Zoey has on her tail. Paul has 10 cubes on his tail. How many cubes does Zoey have on her tail?

Session 10: Understanding the Equal Sign

At A Glance

In this session, children discuss the meaning of the equal sign (*same number as*) regardless of the context. They determine whether equations are true or false by building expressions with linking cubes and comparing. Children determine an unknown number in an equation to make the equation a true statement. They share their reasoning and critique the reasoning of others.

Materials

- Linking cubes
- Blackline master, “Understanding the Equal Sign”

Common Core State Standards focus

Operations and Algebraic Thinking

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.*

1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = \square - 3$, $6 + 6 = \square$.*

MP 3 Construct viable arguments and critique the reasoning of others.

MP 7 Look for and make use of structure.

$$6 = 6$$

$$7 = 8 - 1$$

$$9 - 2 = 5 + 2$$

Activate

All children need 10 linking cubes.

- Say, *“We are going to think about a symbol that we have been using a lot, the equal sign.”*
- *“What is the meaning of the equal sign?”*
- *“Some might say it means ‘the answer is’ or ‘makes.’ For example, they might say that if you have $2 + 4 = 6$, then it means that if you add 2 and 4, the answer is 6. Or 2 plus 4 makes 6.”*
- *“That’s true sometimes, but what is always true about an equal sign?”*
- *“The equal sign always means ‘is the same number as,’ so that’s how I want you to think about it.”*

Record $3 = 3$.

- *“Is 3 the same number as 3?”* (Yes, it is a true equation.)
- *“Prove it with your linking cubes.”*

Record $6 = 2 + 4$.

- *“Is 6 the same number as $2 + 4$?”* (Yes, it is a true equation.)
- *“Prove it with your linking cubes.”*

Record $3 + 1 = 2 + 2$.

- *“Is $3 + 1$ the same number as $2 + 2$?”* (Yes, it is a true equation.)
- *“Prove it with your linking cubes.”*



“So all of these equations are true. Let’s think about what an equation would look like if it were false. Any ideas?”

Record some ideas from children. The following are examples:

- $5 = 6$
“Is 5 the same number as 6?” (No, it is a false equation.)
“Prove it with your linking cubes.”
- $4 = 3 + 3$
“Is 4 the same number as $3 + 3$?” (No, it is a false equation.)
“Prove it with your linking cubes.”
- $2 + 3 = 5 - 1$
“Is $2 + 3$ the same number as $5 - 1$?” (No, it is a false equation.)
“Prove it with your linking cubes.”

Engage

Organize children into pairs. Distribute the blackline, “Understanding the Equal Sign,” and make sure they understand the directions.

- Children work in pairs on Part I.
- To reinforce understanding of the equal sign as meaning “the same number as,” ask children to prove their argument using linking cubes to represent each equation.
- If there is enough time, allow children to begin Part II. They may need guidance when determining the unknown number.
- Children use the linking cubes to build/solve one side of the equation and then use that information to help them find the unknown number on the other side of the equation.

Develop

Organize the class into a whole group.

- Discuss what children discovered in Part I. Allow for an opportunity to prove arguments and critique the reasoning of others.
- Work through Part II together, building with linking cubes to show the relationships.
- Emphasize the equal sign meaning “the same number as.”
- The last equation in Part II has multiple solutions. Continue to allow children to make arguments and critique the reasoning of others.

Understanding the Equal Sign

Part I

Circle any equation that is false.

1. $7 = 7$

2. $3 + 3 = 6$

3. $6 = 4 + 1$

4. $8 = 10 - 2$

5. $5 + 2 = 3 + 4$

6. $8 - 1 = 1 + 6$

7. $2 + 5 = 9 - 1$

8. $8 - 4 = 6 - 2$

Part II

Write a number in each space to make the equation true.

1. $\underline{\quad} = 8$

2. $4 + \underline{\quad} = 5$

3. $6 = 3 + \underline{\quad}$

4. $7 + \underline{\quad} = 5 + 4$

5. $\underline{\quad} - 3 = 4 + 2$

6. $8 + 1 = \underline{\quad} + \underline{\quad}$

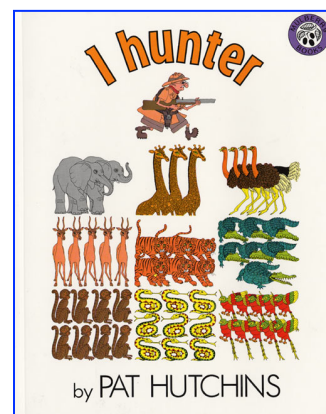
Session 11: What Makes 10?

At A Glance

In this session, children reason about combinations of whole numbers that make 10. They use the book *1 hunter* as a backdrop for analyzing these relationships. Children find groups of linking cubes that make 10, such as 7 and 3. Children see the usefulness of this strategy as they solve a word problem that involves multiple groups of 10.

Materials

- *1 hunter* by Pat Hutchins
- Box of 100 linking cubes



Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.

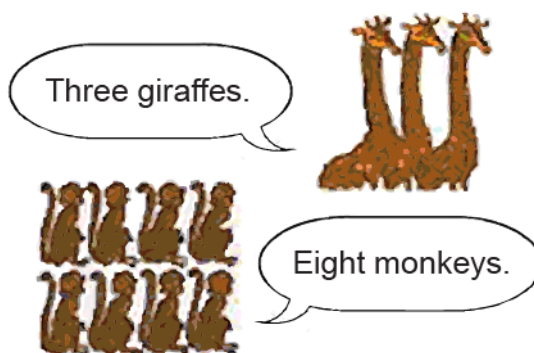
Activate

Introduce and discuss the cover of *1 hunter*.

- Ask, ***“What do you think this book might be about?”***
- ***“How many giraffes are there? What about the monkeys/parrots/antelope/snakes/tigers/ elephants/ostriches/crocodiles?”***

Have children share finding quantities to 10.

- ***“What did you do to figure out the number of giraffes? Did you count or did you just see 3?”***

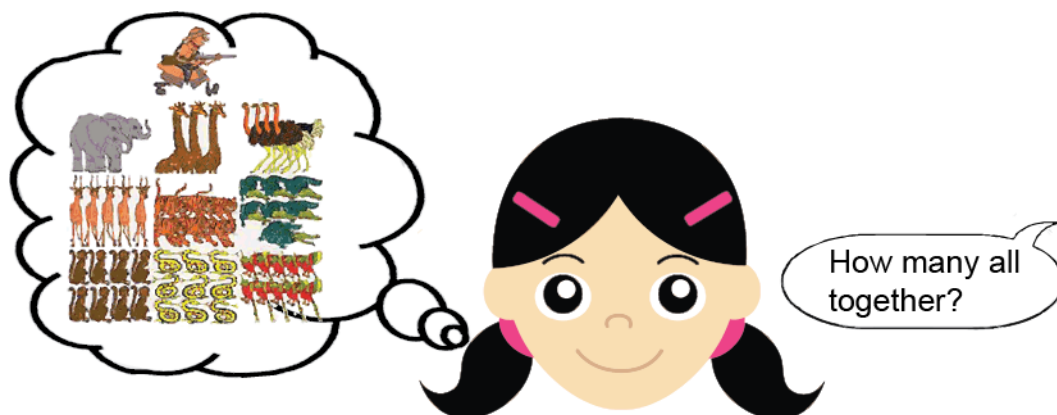


Engage

Read the book to the children. Then, pose the following question:

- ***“How could we go about finding out how many animals, including the hunter, there were altogether?”***

Allow children to try out strategies, using the linking cubes if helpful.



Develop

Now turn to the linking cubes (if you have not already). Ask children to choose a different color set of cubes to represent each group of animals (e.g., brown cubes for the monkeys, yellow cubes for the tigers).

- ***“Make a linking cubes tower to show how many animals there are in each group. Which color do you want to use for the monkeys ... the parrots ... the tigers ... etc.?”***

Draw children’s attention to the tower representing the number of parrots.

- ***“Which group of animals has the most? How many parrots are there?”***

Lead children to see that there are 10 cubes in a complete linking cube tower.

- ***“Did you have to remove any cubes from the tower that shows how many parrots? Why?”***

Lead children to see a pattern in the number of cubes they had to remove from each of the other towers in relation to their original sum of 10.

- ***“How many cubes did you use to represent the number of snakes? How many cubes did you have to take off that tower?”***

Use the tower that represents the parrots as a benchmark for helping guide children to find a way to make a 10 with the tower used to represent the snakes.

- ***“Which group of animals could you add to the snake tower to make a 10 so that it has the same number (same size) as the tower showing the number of parrots?”***

Show the combined snake and hunter tower and record an equation to show the number fact $9 + 1 = 10$.

- ***“The number sentence $9 + 1 = 10$ shows what you did to make a sum of 10. You joined 9 and 1.”***

Encourage children to combine the other towers to make additional sums of 10.

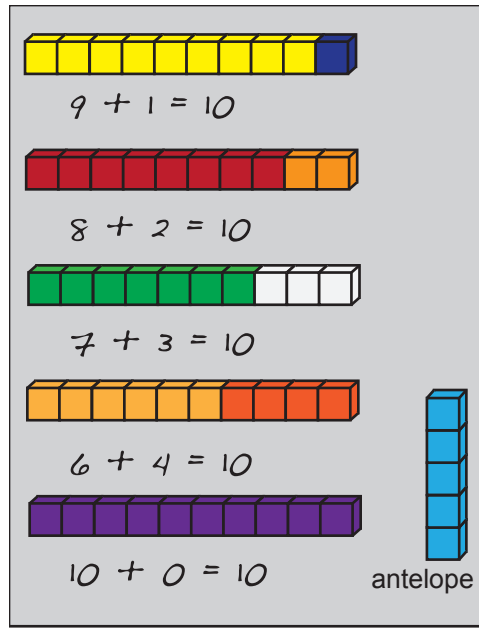
- ***“Now try to combine other groups of animals to make other towers of 10. Place each new tower you make on a sheet of paper, and write a number sentence that matches that combination.”***

Discuss what to do with the only remaining group of animals—the five antelope.

- ***“If we joined another group of animals to the 5 antelope, how many would there need to be in that group to make a 10?”***

Align the towers of 10 and the extra group of 5 in rows and columns. Model how to count by tens for each of the 5 groups of 10 and then by ones for the remaining group of 5. Encourage children to count along with you.

- “Instead of counting each animal one by one, let’s practice counting by tens. Ready, 10, 20, 30, 40, 50 ... now count by ones for the leftovers—51, 52, 53, 54, 55.”



Session 12: Tens-Frame

At A Glance

In this session, children use a tens-frame to help them find combinations that make 10. They show quantities from 1 to 10 on the tens-frame and then find the related number that makes 10. Children find all the combinations that make 10 and then look for patterns among the equations they have made.

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

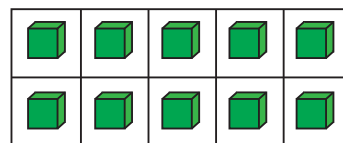
1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.

Materials

- Linking cubes
- Blackline master, “Tens-Frame”
- Blackline master, “Tens-Frame Equation Cards”
- Scissors



Activate

Revisit the book *1 hunter*.

- Ask, ***“How many animals did we find there were altogether in the 1 hunter story?”***

Prompt children to explain how they used the “making a 10” strategy to find the total number of animals in the story.

- ***“Explain how we found that there were 55 animals altogether?”***

Introduce a word problem related to the scenario presented in the book.

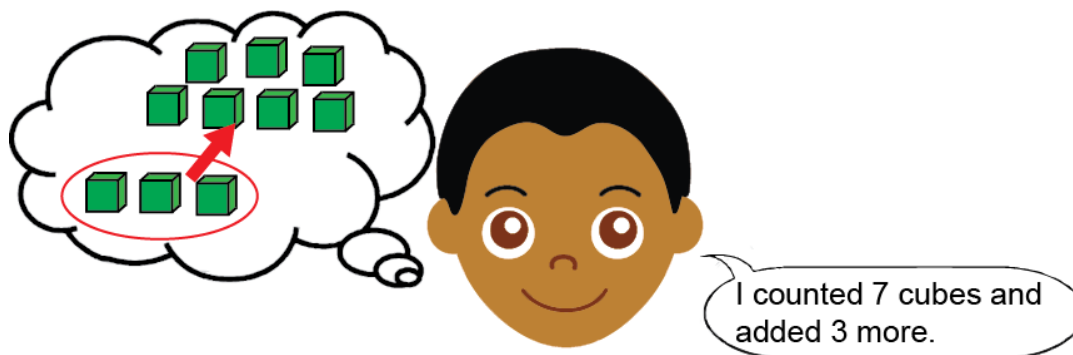
- ***“Let’s say that the hunter actually found the animals hiding in the jungle. To get them to a zoo located across the ocean, he uses a boat that carries up to 10 animals. He puts the 7 crocodiles on the boat first, but realizes that there is room for some more. How many more animals will fit inside? You may use the cubes to help you figure out how many.”***

Children may solve the problem using any one of the following strategies:

- Represent the number of crocodiles with cubes and then continue to add cubes until they get to 10.
- Count on from 7 and keep track of the number of counts (3) to 10: “Seven, eight, nine, ten.”
- Use a number fact (e.g., $7 + 3 = 10$).

Encourage children to explain their strategies.

- ***“How did you figure out how many more animals could get on the boat so that there would be exactly 10 on board?”***



Engage

Give each child a tens-frame and 10 linking cubes. Introduce the tens-frame board.

- ***“This is the layout of the boat the hunter used to carry the animals to the zoo. Each square is a cage, which is room for just one animal.”***
- ***“What do you notice about the squares? How many squares are on the first row? How many are in the second row? How many squares are there altogether? How do you know?”***
- ***“Yes, there are 5 squares on top and 5 squares on bottom—that’s 10 altogether. This picture is called a tens-frame.”***

Count out 10 linking cubes and model how to fill the tens-frame boat. Put one cube in each square as indicated.

- ***“The hunter requires that the animals board the boat in a particular order. He starts on the first row and goes from left to right, putting one animal inside each cage. When the top row is filled, he goes to the bottom row and continues filling each cage one by one, moving left to right.”***

Guide children to use the tens-frame to model the word problem they just solved in the Activate section.

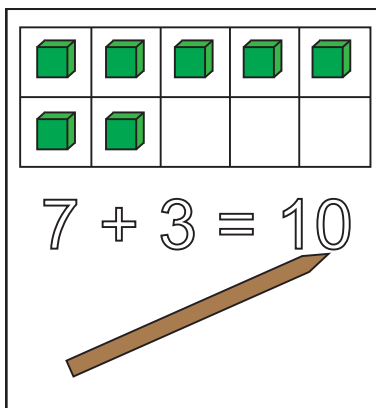
- ***“Use the cubes to show how the hunter would board the 7 crocodiles on the boat.”***

Discuss the arrangement of 7 cubes children have just made on the tens-frame.

- ***“What does the tens-frame show you? How many crocodiles are on the first row? On the second row? How many cages are empty?”***

Model how to write a number sentence to show the number of filled and empty squares on the tens-frame. Write the equation in the large box below the tens-frame.

- ***“The numeral 7 tells how many squares are filled. The numeral 3 tells how many squares are empty. The numeral 10 indicates how many squares there are altogether—filled squares and empty squares.”***



Develop

Give children the two sheets of Tens-Frame Equation Cards and a pair of scissors.

- ***“Cut apart this set of tens-frame cards. When you finish, we will use them to represent all of the different ways to make a 10.”***
- ***“Look at the empty tens-frames on each card you just cut apart. Mark an X inside each square on the tens-frame to show many filled cages there are when 7 crocodiles are on the hunter’s boat. Complete the number sentence in the boxes below. The number you write in the box to the left will tell how many animals are in the boat. The number you write in the box to the right will tell how many empty cages there are left in the boat.”***

Allow children to use the remaining cards to show the other combinations of 10.

- ***“Show what happens when there are 5 antelope on board the boat. How many empty cages are there when there are 10 parrots ... 4 ostriches ... 9 snakes ... 2 elephants ... 8 monkeys ... 3 giraffes ... 6 tigers ... 1 hunter?”***
- ***Which card has the fewest empty cages? Which card would come after that? And then what?***

Make a table (a Combinations of 10 chart) with two columns on a sheet of paper. Label one column *Filled* and the other column *Empty*. Start with the first card that children have ordered from greatest to least or from least to greatest and transfer the numbers that are written in the boxes below the tens-frame to the appropriate columns on the chart.

- ***“When there are 10 cages that are filled, 0 cages are empty.”***

Continue in descending or ascending order transferring the numbers that label each tens-frame card to the appropriate columns on the filled/empty chart.

- ***“When there are 9 cages that are filled, only 1 is empty ... if there are 8 filled, 2 are empty ... etc.”***

Discuss the descending and ascending pattern that unfolds when the numbers that make sums of 10 are paired together in order from greatest to fewest on the chart.

- ***“What do you notice about the numbers in this column? What about the numbers in the other column?”***

Filled	Empty
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

These numbers get smaller going down.

Tens-Frame

Tens-Frame Equation Cards

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Tens-Frame Equation Cards

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Session 13: Composing 10 to Find a Sum

At A Glance

In this session, children connect back to the previous sessions on “making 10” before thinking about sums greater than 10. Children share their own strategy for solving and then use a double tens-frame as a tool for visualizing how to simplify an equation (e.g., converting $9 + 6$ into an easier math fact $10 + 5$) by decomposing the smallest number (6) into two parts (1 and 5) and then taking one of those parts (1) and adding it to the larger number (9) to make a 10 ($9 + 1$). After the teacher models how to use the double tens-frame, children practice the new “composing 10” strategy to solve $8 + 5$.

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

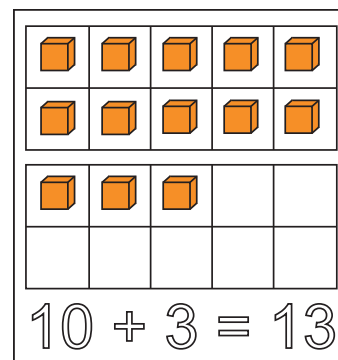
1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.

Materials

- Combinations of 10 chart (developed in Session 12)
- Tens-Frame Equation Cards (completed by children in Session 12)
- Tens-Frame
- Blackline master, “Double Tens-Frame”
- Linking cubes



Activate

Show children the Combinations of Ten chart that you filled out together at the closing of Session 12.

- Ask, ***“What does this list show us? How can we use it to help us figure out combinations that make 10?”***

Lead children to see how the list is useful as a tool for seeing patterns and relationships among numbers.

- ***“How could you use this list of paired numbers to make sure that you have found all the combinations that make a 10?”***

Show children a tens-frame.

- ***“How did you use the tens-frame to find combinations that equal 10?”***

“Combinations of 10”



Filled	Empty
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

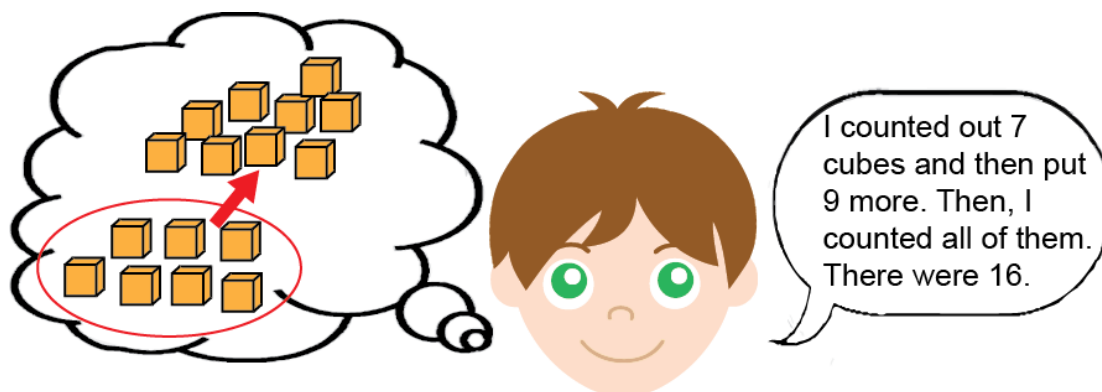
Engage

Introduce the Engage section with the following:

- ***“Listen closely to the following number story.***
On Saturday, Bert picked up 9 shells at the beach. Then, that afternoon, he bought 7 shells at the Shell Shop. How many shells did Bert collect just on that one Saturday?”
- ***“What do you need to do? What is the question asking you to do?”***
- ***“Go ahead and figure out the answer to the problem. You may use cubes or paper and pencil to help you.”***
- ***“How did you find the answer?”***

Responses from children may include the following:

- “I counted out 9 cubes and another set of 7 cubes. Then, I counted all of them—1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16.”
- “I started at 9 (lifts one finger for each count) and said—10, 11, 12, 13, 14, 15, 16.”
- “I did it in my head. I took 1 from the 7 and put it with the 9 to make 10. That left 6, and I know that 10 plus 6 is 16.”



Develop

Introduce the double tens-frame.

- ***“What do you notice about this tool?”***

Guide children to see that there are 20 squares altogether.

- ***“How many cubes could fit on this doubles-ten frame if you put one inside each square? How do you know?”***
- ***“Yes, there are 20 squares—10 on this frame and 10 on this one. First, I am going to model how to use both frames to show two different numbers. Then, I’ll take off just enough from the bottom frame and add it to the top frame to make a 10. Next, I’ll add the 10 with the other number to figure out the total.”***

Use the double tens-frame to represent the two quantities from the problem presented in the Engage section.

- ***“Bert picked up 9 shells at the beach.”*** (Pause to fill the top frame with 9 cubes.)
“He bought 7 shells at the Shell Shop.” (Pause to fill the bottom frame with 7 cubes.)

Take 1 cube from the bottom frame and add it to the 9 cubes in the top frame.

- ***“Now how many cubes are now in the top frame? How many are in the bottom frame?”***

Model a “think-aloud” for a counting-on strategy using the double tens-frame.

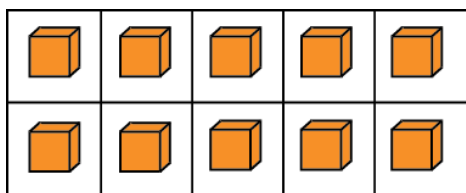
- **“So, if we know there are 10 cubes in the top frame, let’s count on from 10 as I point to each cube in the bottom frame—11, 12, 13, 14, 15, 16.”**

Model a think-aloud for a number fact strategy. Write $10 + 6 = 16$ on a sheet of paper. Explain what each digit in the number 16 tells about the number of cubes in each tens-frame (e.g., one completely filled frame [group] of 10 and 6 single cubes [leftovers].)

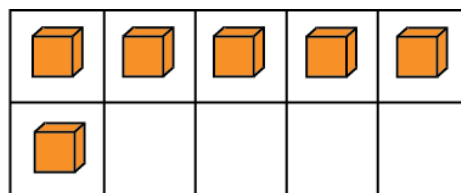
- **“So, if you add 10 and 6, you get 16.”** (Point to the top tens-frame) **“The numeral 1 in the 16 stands for one group of 10, or one completely filled tens-frame.”** (Now point to the bottom frame.) **“The 6 stands for the remaining 6 cubes, which were not enough to fill the bottom frame.”**

Record the number fact $8 + 5$. Allow children to come up with a scenario that matches the equation.

- **“Tell me a story that would use the numbers in this addition fact.”**
- **“What question will we answer?”**
- **“Show me how you can solve the problem you told me using the double tens-frame to make a 10 and a group of leftovers.”**
- **“Tell me what you did. How many cubes did you take from the bottom frame? Why did you take that many? How many cubes are left on the bottom frame?”**
- **“So what is the answer to your word problem?”**



10



11, 12, 13, 14, 15, 16

Double Tens-Frame

Sessions 14 and 15: Practicing Addition Sums to 20

At A Glance

In this session, children solve problems involving 10 and some more. They play a game to practice strategies with the double tens-frame. Children write the corresponding equations and review those equations to see which facts they are becoming more efficient with. This combines two sessions so that children have ample time to play the game and develop their ability to use 10 to help them think about sums greater than 10.

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.

Materials

- Blackline master, “Double Tens-Frame”
- Linking cubes
- Paper and pencil
- Two blank dice (one with numerals 1–6 and the other with numerals 7–10)
- Dot stickers

Prepare ahead of time:

Place one sticker on each face of each die. On one die, write numerals 1–6. On the other die, write 7 and 10 once and 8 and 9 twice on opposite faces.

Activate

Provide each child with a set of up to 20 cubes. Present the following scenario:

- Say, *“Let’s say you have 10 cubes, and I give you 4 more. How many will you have?”*
- *“Arrange the cubes on the double tens-frame. Fill the top frame with 10 and the bottom frame with the remaining 4.”*
- *“Just look at the top frame and without counting tell me how many cubes you put there?”*
- *“And how many are here in the bottom frame?”*
- *“What did you just figure out about adding 4 to 10?”*

Lead children to visualize and describe 14 as 10 and 4.

- *“So, one way to think of 14 is as a group of 10 and 4.”*

Remove the 4 cubes from the bottom frame, leaving the 10 cubes on the top frame intact.

- *“Here is another problem like the one you just solved. Let’s say you have 10 cubes and I give you 6 more. How many would you have? Show on the double tens-frame.”*
- *“How did you figure out there were 16?”*

Pose another “10 and some more” scenario. Assess if children are beginning to understand the “10 and some more” strategy without counting.



Engage

Introduce the Engage section with the following:

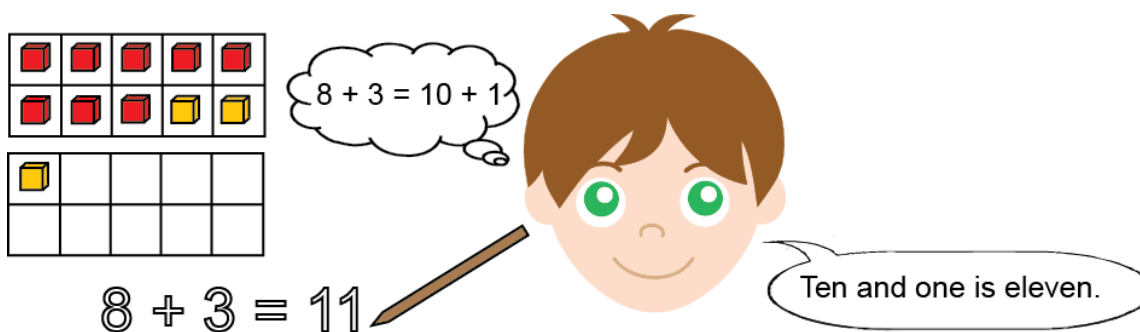
- ***“We will practice adding sums to 20 by playing ‘Make a 10 on the Tens-Frame.’”***

Roll the 7–10 numeral cube, which corresponds to the top tens-frame for this demonstration roll.

- ***“How many cubes should you put on the top tens-frame?”***

Then roll the 1–6 numeral cube, which corresponds to the bottom frame for this demonstration roll.

- ***“How many cubes should you put on the bottom frame?”***
- ***“Write a number sentence that tells how many cubes are in the top frame and the number of cubes in the bottom frame.”***
- ***“How could you use the double tens-frame to make a 10? What would the new number sentence be? What does that tell you about how many cubes are there altogether?”***



Develop

Organize children into pairs. Start this game during Session 14 and continue it through Session 15.

- ***“Partners will take turns rolling both number cubes. One will fill the top frame, and the other will fill the bottom frame. After both frames are filled, write the number sentence on paper. If you have already recorded that combination of numbers, put a check mark beside it. Then, take just enough cubes off the frame that has the most cubes and add them to the frame with fewer cubes to make a 10. Use the ‘10 and some more’ strategy to find the total. Record the total to the left of the equal sign.”***

As children play the game, probe for understanding.

- ***“How can you be sure your answer is correct?”***

After playing several rounds, review children’s list of number sentences. Look for any reversals (e.g., $9 + 2 = 11$ and $2 + 9 = 11$). Guide them to see how both number facts are the same.

- For example, ***“You recorded 9 plus 2 equals 11 here and 2 plus 9 equals 11 here. How are the two number sentences alike, and how are they different?”***

After several rounds, stop and review the list of number facts children have recorded. Have them place a star next to each fact they say is easy to remember.

<p>Step 1</p>	<p>Step 2</p>
<p>Step 3</p> <p>$8 + 3 =$</p>	<p>Step 4</p> <p>$8 + 3 = 11$</p> <p>Ten and one is eleven.</p>

Double Tens-Frame

Session 16: Relating Addition to Subtraction

At A Glance

In this session, children consider the “turnaround” fact strategy when any two numbers in an addition equation are reversed (e.g., $6 + 4$ and $4 + 6$), their sum always remains the same. Children apply this understanding to see how addition facts are related to their subtraction counterparts. They use a tens-frame to develop the “think addition” strategy for solving a problem in which subtraction could be used to figure out a missing number. Children then practice this strategy as they solve a similar word problem.

Common Core State Standards focus

Operations and Algebraic Thinking

Understand and apply properties of operations and the relationship between addition and subtraction.

- 1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

- 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

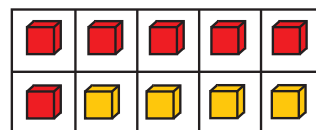
MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.

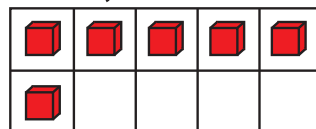
Materials

- Blackline master, “Tens-Frame”
- Linking cubes

$$\text{If } 6 + 4 = 10$$



$$\text{then, } 10 - 4 = 6$$



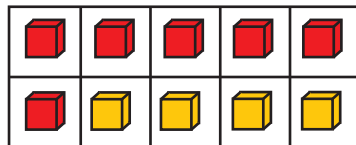
Activate

Display a tens-frame with 6 red cubes (5 in the top row and 1 on the bottom) and 4 yellow cubes (all on the bottom row).

- Ask, ***“How many red cubes do you see? How many yellow cubes? How many cubes are there altogether?”***
- ***“Write a math fact that would show how many red cubes, how many yellow cubes, and the number of cubes altogether.”***

Turn the tens-frame upside down so that the 4 yellow cubes and 1 additional red cube are on the top row and the remaining red cubes are on the bottom row.

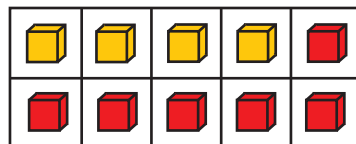
- ***“How many yellow cubes do you see? How many red cubes? How many cubes are there altogether?”***
- ***“Write a math fact that is almost like the one you just wrote but begins with the number that shows how many yellow cubes there are, followed by the number that tells how many red cubes, and then the number of cubes there are altogether.”***
- ***“How are the two math facts alike? How are they different?”*** (They both equal 10, but the numbers 6 and 4 are reversed/in different places.)



$$6 + 4$$

is the same as

$$4 + 6$$



Engage

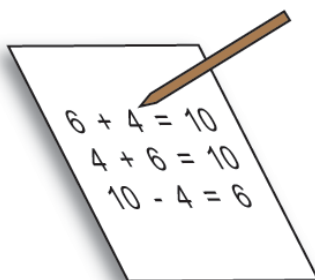
Introduce the new strategy you just modeled for remembering related math facts.

- ***“So, if I know that $6 + 4 = 10$, I also know that $4 + 6 = 10$. That means that once you remember one math fact, you already know its related fact (its reversal). This strategy for remembering math facts is called the ‘turnaround’ strategy because if you just reverse the order of any two numbers, you get the same total (sum).”***
- ***“I am going to make up an addition story that goes with the numbers in the first number sentence you wrote down on paper.”***
- ***“When I reached into my pocket, I found 6 pennies.”*** (Point to the numeral 6 in the number sentence.) ***“Then, I found 4 on the floor.”*** (Point to the numeral 4.) ***“Now I have 10 pennies altogether.”*** (Point to the numeral 10.)
- ***“Now it’s your turn. Make up an addition story that tells about $4 + 6 = 10$.”***
- ***“Now I am going to tell you a story that uses the same numbers you wrote down in both number sentences, but starts out and ends with different numbers:***

I had 10 pennies, but 4 fell down the drain in the sink. Now I only have 6 pennies left.”
- ***“How was the story I just told you different from the other stories you and I made up? How is it similar?”***
- ***“Did the amount of pennies I started out with at the beginning of the story get larger or smaller?”***
- ***“If the plus sign means that you add, which sign do you use to show subtraction?”***
- ***“If you wrote a number sentence to match the subtraction story I just told you, which number would you write first? Then next?”***
- ***“Go ahead and write a number sentence I just told you below the other math facts you wrote down on your sheet of paper.”***

Focus children’s attention on the similarities and differences between the two number sentences $6 + 4 = 10$ and $10 - 4 = 6$.

- ***“What do you notice about these two number sentences?”*** (They have the same numerals [6, 4, and 10] but in a different order.)



Develop

Introduce the “think addition” strategy as a way to solve a related subtraction problem.

- ***“If you are trying to figure out a subtraction problem like 10 minus 4, you can always think addition and ask yourself, ‘What number plus 4 equals 10?’ ”***

Fill the tens-frame board with 10 cubes of any color and show it to children.

- ***“How many cubes are there?”***

Hide the tens-frame behind a sheet of paper so that it is out of children’s view. Remove 5 cubes from the tens-frame and show them to children.

- ***“There were 10 cubes on the board, but then I took off 5. How many are left?”***

Prompt children to practice the “think addition” strategy to solve the subtraction problem you just modeled on the tens-frame.

- ***“Think addition, ‘5 plus what equals 10?’ ”***

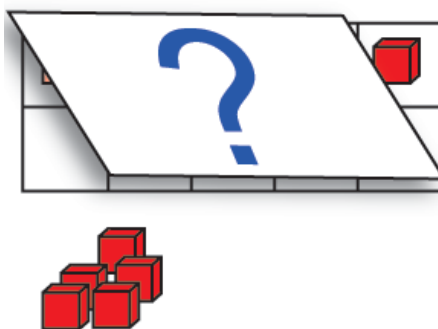
Remove the tens-frame (which now only has 5 cubes) from behind the sheet of paper so that children can verify their answers.

- ***“How many cubes were left on the tens-frame?”***

Write $5 + 5 = 10$ and $10 - 5 = 5$ side by side on a sheet of paper. Explain how they are related.

- ***“So, if I know that 5 plus 5 is 10, then I know that 10 minus 5 is 5.”***
- ***“Here is a new word problem. Think about what addition fact you could use to help you solve it. You can use cubes if you need them.”***

Sammy had 9 grapes. Then he ate some of them for lunch. Now he has only 6 grapes left. How many grapes does he eat for lunch?”



Tens-Frame

Session 17: Subtraction as “Think Addition”

At A Glance

In this session, children become more familiar with math facts that make 10, working on recall through 10. Then children experience an activity called “Say a 10 Fact” in which they fill a tens-frame card with a partial set of cubes to find out how many more are needed to make one group of 10. The purpose of the activity is to lead children to use addition to solve subtraction problems.

Materials

- Blackline master, “Tens-Frame Fact Cards,” that have been *cut apart ahead of time*
- Blackline master, “Tens-Frame”
- Linking cubes

Common Core State Standards focus

Operations and Algebraic Thinking

Understand and apply properties of operations and the relationship between addition and subtraction.

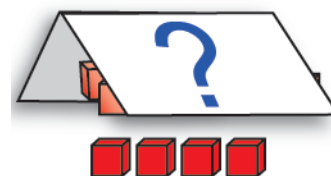
- 1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

- 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.



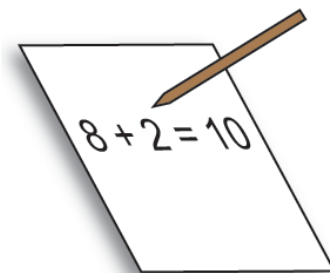
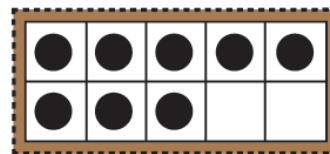
Activate

Show children a tens-frame fact card with 8 dots.

- Ask, *“How many dots do you see? How many more do you need to make a 10?”*

Record $8 + 2 = 10$.

- *“8 plus 2 equals 10 is a math fact that goes with this card and is one way to show that there are 8 dots on the tens-frame, that 2 squares are empty, and that there are 10 squares altogether.”*



Show children a tens-frame fact card with 6 dots.

- *“What is the 10 fact for this card?”*

Show children a tens-frame fact card with 3 dots.

- *“What is the 10 fact for this card?”*

Engage

Show children a blank tens-frame.

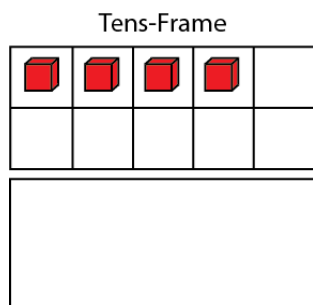
- *“Now we are going to play a game called, ‘Say a 10 Fact.’ I will say a number and then you will use cubes to show that number on the tens-frame. Then, you will say the 10 fact to complete the number sentence.”*

Make a set of cubes available for children to use to show each number you say.

- *“Four.”*

Continue saying other numbers. If children continue to rely on counting to figure out the 10 fact, restate the “what number plus another number equals 10” question to lead children to begin recalling number facts from memory.

- For example, *“Five. Nine. Two.”* (If necessary, ask, *“What number plus 5/9/2 equals 10?”*)



Four plus six equals ten.

Develop

Count out 10 linking cubes. Then, cover all 10 with a sheet of opaque paper.

- ***“How many cubes did I count? How many did I hide?”***

Count and remove 5 cubes from under the sheet of paper. Keep these in view of children.

- ***“How many cubes did I take out? How many are still hiding? Think, ‘5 and what makes 10?’ ”***
- ***“So if 5 plus 5 is 10, then what is 10 minus 5?”*** If children fail to respond, say, ***“There were 10 under the sheet of paper, and I took out 5, so 5 are now hiding. Another way to say that is ‘10 minus 5 is 5’ or ‘10 take away 5 is 5.’ ”***

Uncover the cubes and instruct children to write a subtraction fact that matches the hiding scenario you just modeled.

- ***“How do you write the subtraction fact 10 minus 5 is 5?”***

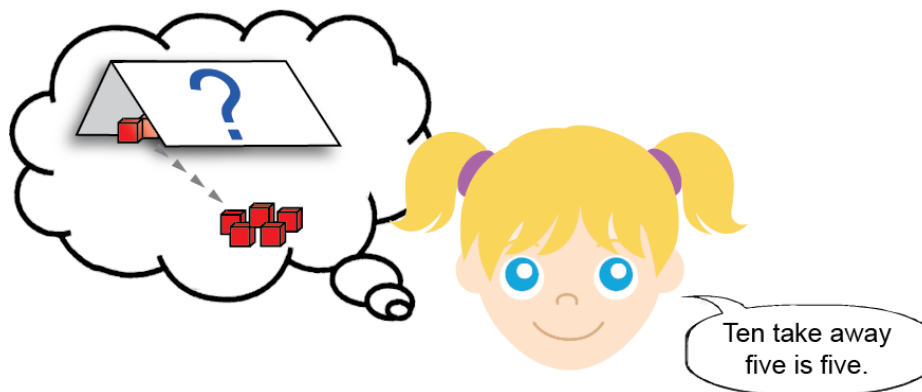
Count out 12 cubes and cover them with the sheet of construction paper. (Allow children to count the 12 cubes before you hide them so that they can confirm that there are indeed 12.) Then, count and remove 5 from underneath the sheet, keeping them within children’s view.

- ***“Think, ‘Five and what make 12?’ ”***

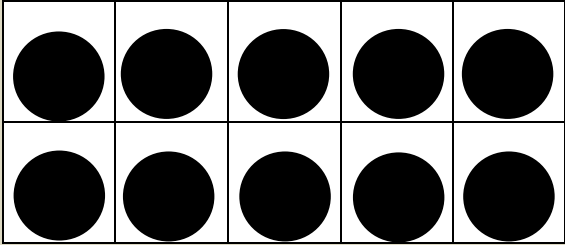
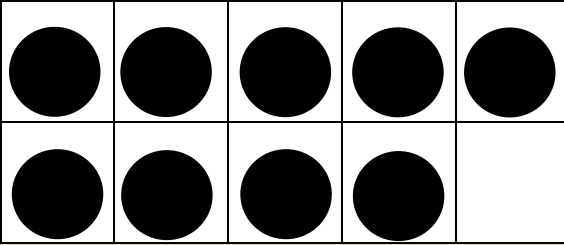
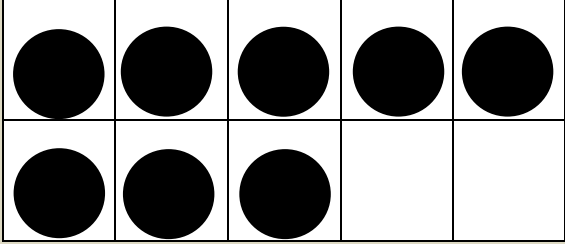
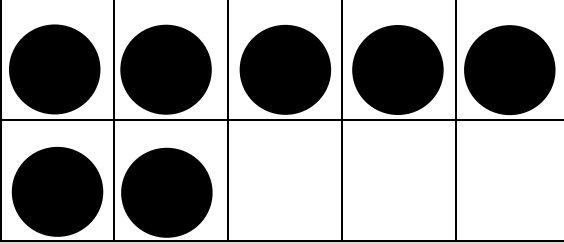
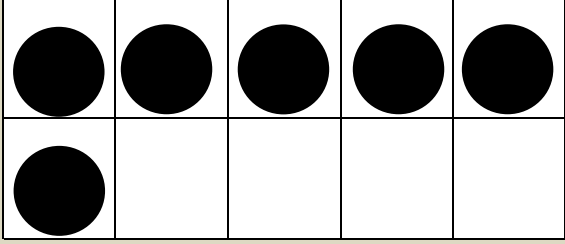
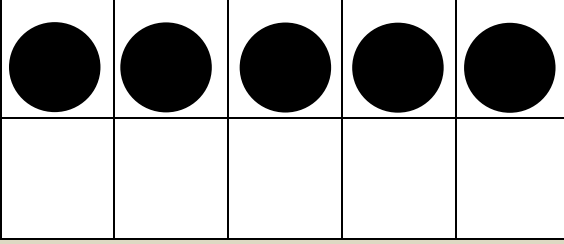
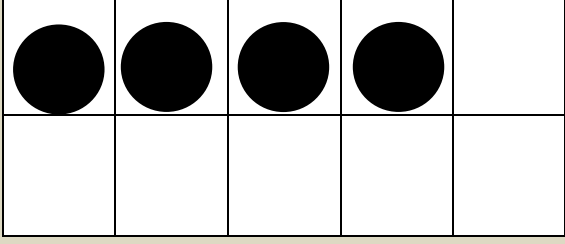
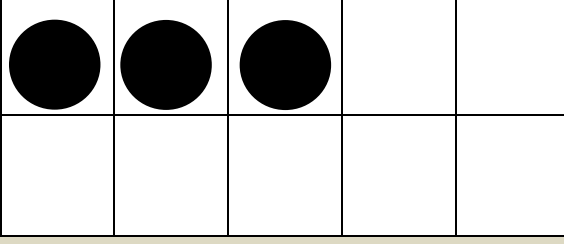
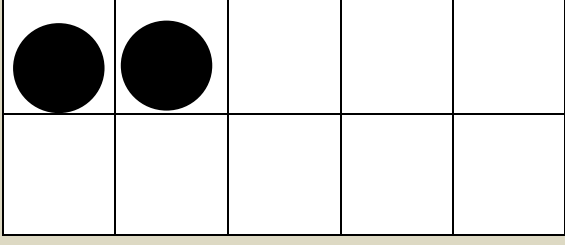
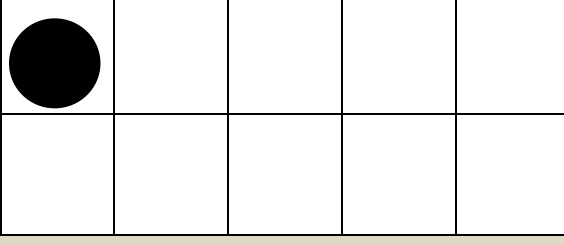
Uncover the remaining cubes.

- ***“What number fact can you write to show 12 minus 5 is 7?”***

Continue with other problems as time permits.



Tens-Frame Fact Cards *(cut this apart ahead of time)*

Tens-Frame

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Session 18: Building Up Through 10 to Add and Subtract

At A Glance

In this session, children recall related math facts to 10 using the tens-frame as a visual model. This continues to emphasize the relationship between addition and subtraction and how that relationship can be used to solve problems. Children use a double tens-frame and a set of cubes to practice the “build up through 10” strategy. The focus is on using the new strategy to figure out the difference between a larger number that the teacher calls out and a smaller number, which children show on the tens-frame.

Materials

- Tens-Frame Fact Cards (used in Session 17)
- Linking cubes
- Blackline master, “Tens-Frame”
- Blackline master, “Double Tens-Frame”

Common Core State Standards focus

Operations and Algebraic Thinking

Understand and apply properties of operations and the relationship between addition and subtraction.

- 1.OA.4 Understand subtraction as an unknown-addend problem.
For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.

Add and subtract within 20.

- 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.



Activate

Show children a 1-dot tens-frame fact card.

- ***“What is the 10 fact for this card? What number plus 1 is 10? How many more dots do you need to reach 10?”***

Record the math fact $9 + 1 = 10$ on a sheet of paper. Then, right below it, write $10 - 1 = \triangle$.

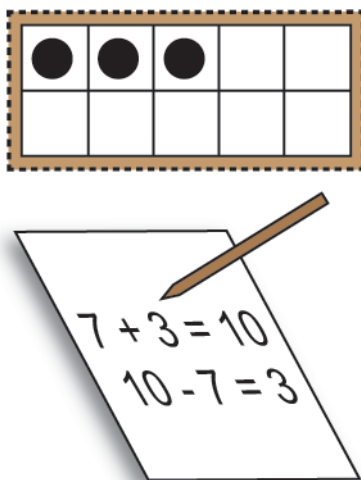
- ***“How do you read the first math fact? What is the second number sentence asking you to figure out? How can you use what you know about ‘1 plus 9 equals 10’ to figure out ‘10 minus 1’?”***
- ***“How are these two math facts the same? How are they different?”***

Show children a tens-frame fact card with 3 dots.

- ***“What is the 10 fact for this card? What plus 3 makes 10?”***

Record $3 + 7 = 10$.

- ***“What subtraction fact could you use to solve 3 plus 7 equals 10?”***



Engage

Use a double tens-frame and cubes to model an addition problem in which making a 10 is a useful strategy for determining the sum/answer.

- ***“Annette had 9 crayons.”*** (Place 9 cubes in the top frame.) ***“Her teacher gave her 5 more crayons.”*** (Place 5 cubes in the bottom frame.) ***“How many crayons does Annette have now?”***
- ***“What math fact would you write to show how you joined 9 and 5 to make 14?”***

Record $9 + 5 = 14$.

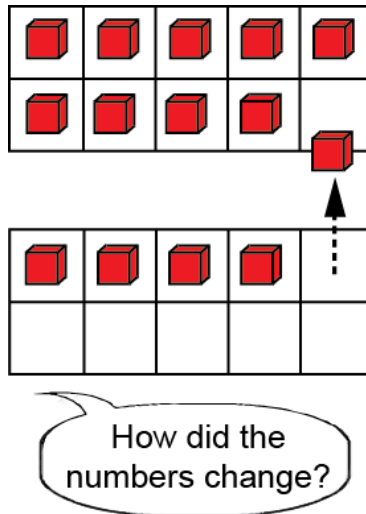
- ***“How many cubes would I need to take from the bottom frame and add to the top frame to change 9 to 10?”***

Move 1 cube from the bottom frame to the top frame. Lead children to see that as 1 cube was removed from the bottom frame to *increase* the 9 cubes in the ten frame to 10, the number in the bottom frame (5) *decreased* by 1.

- ***“What happened to the number of cubes in the bottom frame when I moved 1 cube to join the 9? Are there still 5?”***
- ***“If we break apart 5 into 4 and 1, we can join the 1 with the 9 to make 10, which leaves the 4 by itself.”***

Discuss why making a 10 is a useful strategy for solving an addition problem.

- ***“What did I do to make the problem easier to solve? Which fact is easier to remember—9 plus 5, or 10 plus 4? Why?”***



Develop

Introduce the Develop section with the following:

- ***“Make a number line that starts at 1 and ends at 10. Which number is only 1 down from 10? How far is 8 from 10?”***
- ***“Since 9 and 8 are so close to 10, it is easier to change them into a 10 when you are adding them to other numbers. Today we are going to practice the ‘building up through 10’ strategy with math facts that begin with 8 and 9.”***

Show children a tens-frame fact card with 9 dots. Provide children with a blank tens-frame and 10 cubes to model the “building up through 10” strategy.

- ***“What number do we combine with 9 to get 11? You can use the cubes and the blank tens-frame card to help you figure out ‘what plus 9 gets you up to 11’.”***

Lead children to relate the addition procedure you modeled to subtraction.

- ***“11 minus what gets you down to 9?”***
- ***“So if 9 is 1 down from 10, and 10 is 1 down from 11, I can write a subtraction fact 11 minus 2 is 9.”***

Show children a tens-frame fact card with 8 dots. Once again, allow children to use cubes and the blank tens-frame to show $8 + \triangle = 11$.

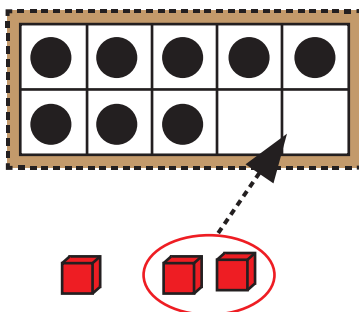
- ***“How can we make 11 starting with 8 in the tens-frame?”***
- ***“11 minus what gets you down to 8?”***

Record the number sentence $11 - 3 = 8$ below $11 - 2 = 9$.

- ***“If I join the 2 that I added to 8 and the 1 that I added to 10 together, I get 3. That means 11 minus 3 is 8.”***

Continue practicing the “building up through 10” strategy with other problems as time allows.

- ***“How can we make 13 starting with 8? 14 starting with 9?”*** (etc.)



Tens-Frame

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Double Tens-Frame

Session 19: Backing Down Through 10 to Subtract

At A Glance

In this session, children solve a problem in which one part of a quantity is unknown (e.g., $17 = 8 + \triangle$). They are introduced to the “back down through 10” strategy to subtract. This strategy is useful for finding the answer to subtraction problems in which there is a difference of 8 or 9. Children have an opportunity to practice this strategy through the use of the tens-frame.

Materials

- Blackline master, “Double Tens-Frame”
- Blackline master, “Tens-Frame”
- Linking cubes

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.



Activate

Introduce the Activate section with the following:

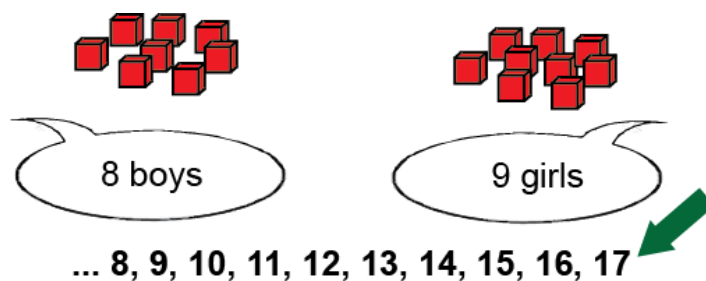
- Say, ***“Today we will begin with a problem to solve. You may use any tools you need to solve the problem.”***

Introduce a problem in which children can use subtraction to figure out a missing quantity that is part of a larger quantity (e.g., a part of one whole).

- ***“There are 17 children swimming in the pool. 8 of the swimmers are boys. The rest of the children are girls. How many girls are swimming?”***

Assess what children understand about the structure and the numbers within the story problem you just presented. Repeat the problem if necessary.

- ***“What do you know about the problem?”***
- ***“What is the question?”***
- ***“You may use cubes, a tens-frame, a number line, or pencil and paper to help you find the solution to the word problem.”***



Engage

Record the equation $17 - 8 = 9$ to show what children did (i.e., subtract) to solve the problem. Restate what children did as you record.

- For example, ***“So, I see you started with 17 and then subtracted 8, which gives you 9.”***

Arrange 17 cubes on a double tens-frame to show one group of 10 and 7 leftovers. Emphasize how the cubes are arranged on the double tens-frame to show a group of 10 and 7. Then, refer to a number line as you compare the position of the digit 7 in the number 17 to the number 8 that follows the minus sign. Point to each digit as you compare.

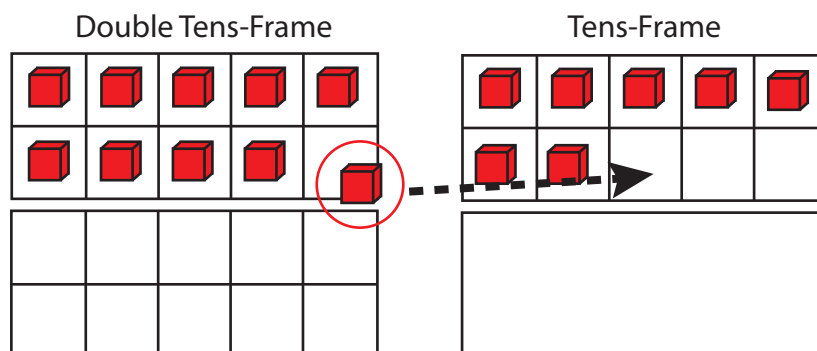
- ***“I noticed that the 7, which is in the ones place, is only 1 away from 8 on the number line.”***

Refer to the 17 cubes on the double tens-frame. Remove the 7 leftover cubes from the bottom frame and place them on the single tens-frame. Describe what you are doing to make the process and reasoning behind the “back down through 10” strategy more explicit to children.

- ***“When I subtract, I am going to leave the 10 cubes on the top frame where they are and just remove all of the 7 leftover cubes that are on the bottom frame. That will get me ‘back down’ to 10.”***

Take 1 cube from the group of 10 that is arranged on the top frame of the double tens-frame board and put it with the 7 cubes that you just placed on the single tens-frame. Lead children to see that you have now subtracted 8 cubes from the original 17.

- ***“So far, I have only removed 7 cubes from the 17. I need to subtract 8. So, I’ll take 1 cube from the 10 and add it to the 7 on the other frame. How many are now left on the double tens-frame?”*** (9)



Develop

Introduce the Develop section with the following:

- ***“Now it is your turn to practice the new strategy that I just modeled. It is called the ‘back down through 10’ strategy because you try to make a 10 when you subtract.”***

Write the number sentence $14 - 6$ in vertical format.

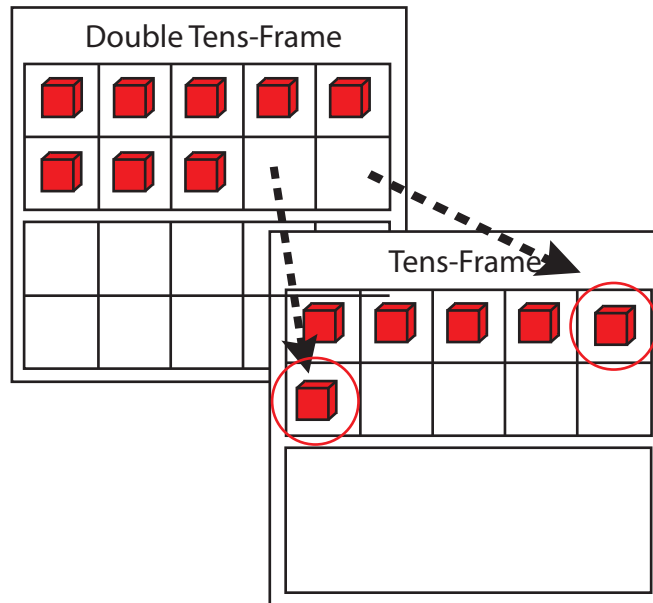
- ***“Use the double tens-frame to show 14. Think about the 4 in the ones place and how far/close it is from 6 if you were looking at the two digits on a number line.”***
- ***“What will you do next?”***
- ***“You had 14 cubes and removed 4. What will you do now?”***
- ***“How many cubes are left? How would you write that as a number fact?”***

Ask children to continue to use the “back down through 10” strategy. Select math facts from the list below:

$17 - 9 =$	$14 - 5 =$	$18 - 9 =$	$13 - 4 =$
$16 - 7 =$	$12 - 3 =$	$16 - 8 =$	$12 - 4 =$
$15 - 6 =$	$11 - 2 =$	$15 - 7 =$	$11 - 3 =$

Do as many problems as time allows.

- **“Solve** [see the list of subtraction facts in the previous list] **using the ‘back down through 10’ strategy.”**



Tens-Frame

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Double Tens-Frame

Session 20: Reviewing Strategies

At A Glance

In this session, children have an opportunity to practice the six strategies they have been focusing on since Session 11: counting on, making a 10, turnaround facts, thinking addition, building up through 10, and backing down through 10. Children compare three numbers to determine how they are related. Children engage in a Missing Number Activity Sheet designed to deepen their understanding of the strategies they have been using and the relationship between addition and subtraction.

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).

MP 2 Reason abstractly and quantitatively.

MP 7 Look for and make use of structure.

Materials

- Linking cubes
- Tens-Frame
- Double Tens-Frame
- Blackline master, “Missing Number Activity Sheet”
- Blackline master, “Addition and Subtraction Strategies”



Activate

Provide materials for children to model or show thinking for solving a word problem. Observe the strategies they use.

- ***“Think about how you can solve the following problem using any of the six strategies we have learned up to this point.***

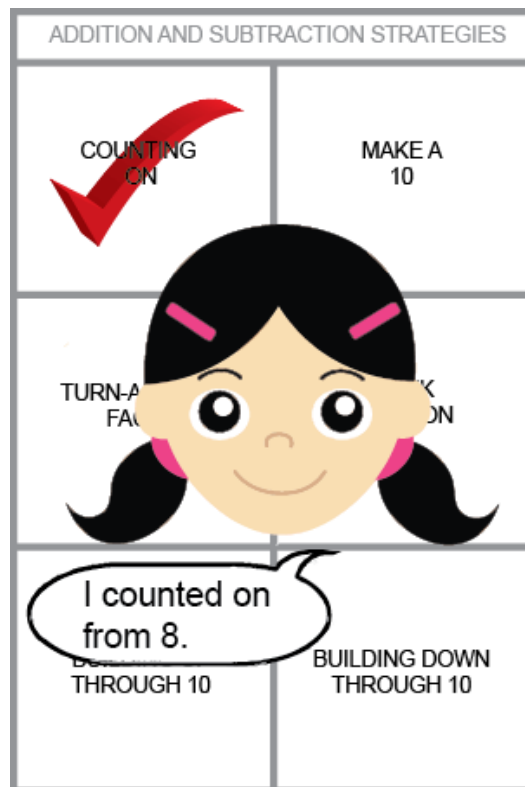
Joe had 8 pennies. His mother gave him some more pennies. Now he has 12. How many pennies did his mother give him?”

Display the Addition and Subtraction Strategies list, which shows the six strategies introduced in this series of sessions.

- ***“You have learned six strategies in our lessons: counting on, making a 10, turnaround facts, thinking addition, building up through 10, and backing down through 10.”***

Allow children to review the list of strategies to determine if they used any of them to solve the word problem presented.

- ***“Did you use any of these strategies to help you solve the problem you just worked? How?”***



Engage

Use a tens-frame, a double tens-frame, and cubes to model other strategies children did not use.

- *Turnaround facts*
“If 8 plus 4 is 12, then 4 plus 8 is 12. It doesn’t matter what number I add first because the answer is the same.”
- *Making a 10*
“If we break apart the 4 and put 2 with the 8, we can make a 10. Then, 10 plus the extra 2 is 12.”
- *Thinking addition*
“If I know what plus 8 makes 12, which is 4, then I do not have to subtract.”
- *Building up through 10*
“If I know that 2 plus 8 is 10, I will have 2 extra, which gets me to 12.”
- *Backing down through 10*
“If I know that 12 minus 2 is 10, then I can go down 2 from 10, which gets me to 8.”

Record the numbers 8, 14, and 6 horizontally. Guide children to see how the numbers are related.

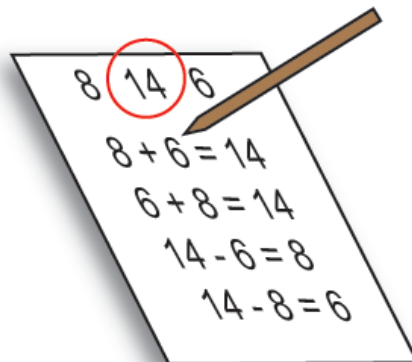
- ***“Why do these numbers belong together? If you add 6 to 8, what happens? What happens if you subtract 8 from 14?”***

Circle the 14.

- ***“Why did I circle the 14?”***

Write a series of equations using these numbers ($8 + 6 = 14$, $6 + 8 = 14$, $14 - 6 = 8$, $14 - 8 = 6$). Explain how they are related.

- ***“When you add the smaller numbers—the 6 to the 8 or the 8 to the 6—you always get 14. So, whenever you subtract the smaller numbers from 14, you get the number you did not take from 14—the 6 or the 8—as your answer. The number sentences I wrote are like a family of facts because they are related to each other.”***



Develop

Display the Missing Number Activity Sheet. (You can also cut this sheet apart into a set of cards.)

- ***“Today we will look at some Missing Number cards and try to figure out what number is missing.”***

Show children the Missing Number card with the circled 10, an empty box, and a 7.

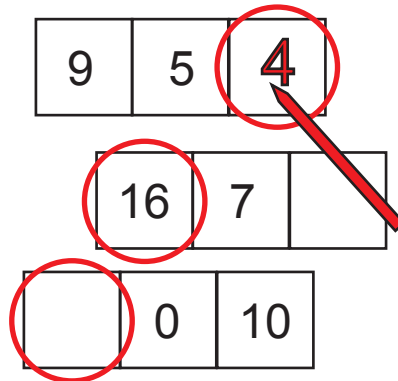
- ***“Which number is missing?”***

Ask children to explain their thinking and then write down the math fact.

- ***“How do you know that 3 is the missing number?”***

Have children complete as many of the remaining cards as time allows. If they fill in a card with an incorrect number, allow them to use cubes to model how to find the missing number and then explain their thinking.

- ***“What number is missing? How do you know? Show me your thinking with these cubes.”***



Missing Number Activity Sheet (may be cut into cards)

10		7	9	5			13	6
	9	18		4	7	8	15	
2		8	16	7			3	9
6	12		8	5			0	10
14		5	8	12		7	5	
6		11	5	10			8	10

Addition and Subtraction Strategies

<p style="text-align: center;">Counting On</p> <p>8/ ... 9 10 11 12</p> <p style="margin-left: 20px;">⇒ ○1 ○2 ○3 ○4</p> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; text-align: center;">Eight</div> </div>	<p style="text-align: center;">Making a 10</p> <div style="text-align: center; margin-bottom: 20px;"> </div> <div style="text-align: center;"> </div>
<p style="text-align: center;">Turnaround Facts</p> <div style="text-align: center; margin-top: 20px;"> </div>	<p style="text-align: center;">Thinking Addition</p> <div style="text-align: center; margin-top: 20px;"> </div> <div style="text-align: center; margin-top: 20px;"> </div> <p style="text-align: center; font-size: 2em; margin-top: 10px;">12 - 4 = ?</p>
<p style="text-align: center;">Building Up Through 10</p> <div style="text-align: center; margin-top: 20px;"> <p style="font-size: 2em;">10</p> <p style="font-size: 2em;">↑</p> <p>9 + 1 = 10</p> <p style="font-size: 2em;">↑</p> <p>8 + 2 = 10</p> </div>	<p style="text-align: center;">Backing Down Through 10</p> <div style="text-align: center; margin-top: 20px;"> <p>12 - 2 = 10</p> <p style="font-size: 2em;">↓</p> <p>11 - 1 = 10</p> <p style="font-size: 2em;">↓</p> <p style="font-size: 2em;">10</p> </div>